Application of Spectral Collocation Method to Conduction and Laminar Forced Heat Convection in Eccentric Annuli

Woo Gun Sim* and Jong Min Kim*

(Received June 3, 1995)

Numerical approach based on the spectral collocation method has been utilized for analyzing heat convection and conduction in eccentric annuli. An eccentric instead of concentric annular duct is sometimes used as a fluid-flow and heat-transfer device especially in nuclear power plants. The hydrodynamically and thermally fully developed laminar flow with uniform heat flux through the inner and outer walls has been analyzed. Also, the conductive heat transfer problem, with uniform rate of internal heat generation in long hollow cylinder, has been solved. The governing equation for the present analysis is Poisson's equation with constant non-homogeneous term. Considering temperature and velocity distributions in eccentric annuli, Nusselt numbers and wall shear stresses are presented for various range of eccentricities. The spectral collocation method used in this study is verified by comparing the numerical solutions from the existing analytical solution and it is clear that this method is appropriate for assessing a more complicated heat transfer problem.

Key Words: Spectral Collocation Method, Uniform Heat Flux, Viscous Shear Stress, Friction Factor, Mixed Mean Temperature, Nusselt Number

Nomenclature

a : Radius of inner cylinder
b : Radius of outer cylinder
c : Specific heat
$C_j(Z)$: Chebyshev polynominals
D_h : Hydraulic diameter $(=2G)$
e : Eccentricity
$F_k(\theta)$: Fourier function
G : Annular gap $(=bg^+)$
h : Heat transfer coefficient
k : Thermal conductivity
Nu : Nusselt number $(=hD_h/k)$
$\dot{q}$: Heat generated per unit volume
$\dot{q}'$: Heat energy transferred through unit length per unit time
$\dot{q}''$: Heat flux
Re : Reynolds number $(=\rho u_m D_h/\mu)$
t^+ : Dimensionless temperature $(=(t_w-t)/(t_w-t_m))$
t_w : Wall temperature
u : Axial flow velocity
Z : Radial coordinates in computational domain

Greek Letters

α : Thermal diffusivity $(=k/\rho c)$
λ : Friction factor
μ : Dynamic viscosity
ρ : Density of fluid
τ : Shear stress

Superscripts

$+$: Stands for nondimensional parameter
$-$: Stands for average value

Subscripts

c : Stands for concentric case
i : Refers to inner cylinder
j : Order of Chebyshev polynomials
k : Order of Fourier function
m : Stands for mean or mixed mean value
o : Refers to outer cylinder

* Korea Atomic Energy Research Institute

1. Introduction

A considerable interest in the problems of flow and heat transfer in annuli, both concentric and eccentric, has arisen in recent decades (Reynolds, *et. al*., 1963 ; Cheng and Hwang, 1968 ; Trombetta, 1971). Eccentric annuli are employed in a variety of heat transfer systems especially in nuclear power plants. The eccentricity may stem from design constraints, but more frequently it is a result of manufacturing tolerances, misalignment and deformation in service for the nominally concentric annular configuration ; *e. g.*, between guide tube and control rod in pressurized water reactor and between pressure and calandria tubes in pressurized heavy water reactor. An engineered thermal switch with eccentric configurations was proposed to increase the heat transfer in the gap under accident conditions compared with normal operation (Chang, *et. al.*, 1994). In this eccentric case, the heat transfer problem becomes more complicated as compared to the concentric case. Thus, there is a large number of significant interest on the heat transfer in eccentric annuli.

Exact analytical solutions for fully developed laminar flow in eccentric annuli were presented by Piercy *et. al*. (1933) and Snyder & Goldstein (1965), based on bipolar transformation proposed by Macdonald (1893) for torsion moment of an eccentric annulus. The forced convection to hydrodynamically and thermally fully developed laminar flow in eccentric annuli was studied by Trombetta (1971) analytically. An exact theoretical solution for heat conduction in an eccentrically hollow long cylinder with internal heat generation was presented by El-Saden (1961). However, for more complex engineering configurations, involving complicated thermal boundary condition, steep configuration and diffuser in the annular passage, analytical solutions do not exist sometimes. Thus, it is obviously required to develop numerical approach instead.

A comprehensive research effort for eccentric configuration has been initiated by the leading author, aiming at unsteady flow solution to be eventually used in the study of the dynamics and stability of various systems (Sim and Cho, 1993 ; Mateescu, *et. al.*, 1994 ; Sim, *et. al.*, 1995). Suitable spectral expansions for the fluid-dynamic parameters were used, such as Chebyshev polynomials and Fourier functions of the spatial expansions, as well as complex exponential functions of time and frequency. A brief review on the spectral method was shown in the reference (Mateescu, *et. al.*, 1994).

In most of the recent spectral method to flow problem, the spectral expansions for spatial discretization are used in conjunction with a temporal discretization, using a time splitting method based on finite-difference approach (Marcus, 1984 a & b). Fractional time steps with time splitting method were used to solve the Taylor-Couette flow problems by computing separately the pressure and viscous contributions. In contrast to the spectral methods using a finite-difference time discretization, the present spectral method is fully spectral approach with only spatial expansions based on collocation method. The present spectral method was found to have excellent accuracy and computing efficiency and can be implemented on any personal computer *vis-a-vis* the spectral method using a time splitting approach which was implemented, for example, on a CRAY-1.

The initial motivation of this paper is to develop a numerical approach with sufficiently accurate solution, without using the bipolar coordination (Trombetta, 1971 ; El-Saden, 1961), based on a spectral collocation method. Using the numerical method, hydrodynamically and thermally fully developed laminar flow, with uniform heat flux through thermal boundaries in eccentric annuli, has been studied. Eventually, this numerical method will be extended for more complicated problems, of which analytical solutions do not exist, as future work. The steady temperature distribution, in an infinitely long eccentrically hollow cylinder with uniform rate of internal heat generation, has been evaluated. To verify the present spectral collocation method on the heat transfer problem, the numerical solutions are compared to the existing analytical solutions (Cheng and Hwang, 1968 ; Snyder and

Goldstein, 1965).

In the present study, thermal and hydrodynamic parameters are expanded in the terms of Chebyshev polynomial for radial coordinate and Fourier function for circumferential coordinate. The problem is solved in computational domain with aid of coordinate transformation. Governing equation and boundary conditions are expressed in matrix form, applying the equations on equally distributed collocation points or wall surface. The unknown coefficients defined in the parameters are determined from the matrix form of the algebraic equations by Gauss Seidel iteration method based on pivot point.

2. Spectral Collocation Method Formulation

The governing equations of a convective heat transfer problem, represented by the Navier-Stokes and energy equations, may be expressed in general matrix form as

$$\boldsymbol{E}\left(\frac{\partial \boldsymbol{f}}{\partial x_1}, \frac{\partial \boldsymbol{f}}{\partial x_2}, \cdots, \frac{\partial^2 \boldsymbol{f}}{\partial x_1^2}, \frac{\partial^2 \boldsymbol{f}}{\partial x_2^2}, \cdots, (\boldsymbol{f}\cdot\nabla)\,\boldsymbol{f}, \cdots, x_1, x_2, \cdots\right)=0 \quad (1)$$

where x_1, x_2, $\cdots$ are independent variables representing geometrical coordinates and $\boldsymbol{f}=(f_1, f_2, \cdots, f_N)^T$ is the vector of thermal and hydrodynamic parameters, such as the velocity components and temperature. The above governing equations are subjected to specific boundary conditions, forming in general N_b algebraic or differential equations ;

$$\begin{aligned} &E_{N+p}(f_1, f_2, \cdots, x_1, x_2, \cdots) \\ &=B_p(f_1, f_2, \cdots, x_1, x_2, \cdots) \\ &p=1, 2, \cdots, N_b \end{aligned} \quad (2)$$

The present spectral collocation method is based on suitable spatial expansion for f_n, using concomitantly Chebyshev polynomial $T_j(x_1)$ and Fourier functions $F_k(x_2)$, such as sine and/or cosine functions. For example, the following type of expansion is used for the thermal and hydrodynamic parameters in a two dimensional heat transfer problem ;

$$f_q(x_1, x_2)=\sum_j\sum_k \widehat{F}_{q:jk} T_j(x_1)F_k(x_2) \quad (3)$$

where $\widehat{F}_{q:jk}$ are a priori unknown coefficients. The spatial expansion is usually performed in the computational domain, obtained by a convenient transformation from the physical domain, when a more complex geometry involved, such as in the case of eccentric annular configurations. The a priori unknown coefficients $\widehat{F}_{q:jk}$ are determined in the present method using a collocation approach, by which the governing equations and boundary conditions are rigorously satisfied at a number of specified collocation points within computational domain and on its boundaries. In the mathematical sense, the collocation approach belongs to the class of methods of weighted residual for the discretization of differential equations, in which the test functions are represented by translated Dirac delta functions, centered at the collocation points.

As a result, the residuals of the governing equations and boundary conditions, obtained by using truncated expansions of the form (3), are set to zero at specified collocation points. This leads in the case of expansions (3) to algebraic systems of equation, obtained from Eqs. (1) and (2) in the form

$$\begin{aligned} &\widehat{E}_s(\widehat{F}_{1:11}, \cdots, \widehat{F}_{q:jk}, \cdots, \widehat{F}_{N:M_JM_K}\; x_{1J}\;, x_{2K}) \\ &=0 \\ &s\in\{1, N+N_b\},\; J\in\{1, M_J\} \\ &K\in\{1, M_K\} \end{aligned} \quad (4)$$

where (x_{1J}, x_{2K}) are the coordinates of the collocation points. This system of equations is then solved for the unknown coefficients $\widehat{F}_{q:jk}$ of the spectral expansions of the fluid parameters.

3. Numerical Formulation of the Governing Equations

The fundamental study of the heat transfer is performed for a hollow cylinder with uniform heat sources, and then convective heat transfer problem is treated. The governing equation for steady-state annular flow can be obtained from Navier-Stokes equation, considering both axial flow velocity and axial pressure gradient. In the energy equation, the viscous dissipation effect is neglected. The inner and outer cylinder radii are

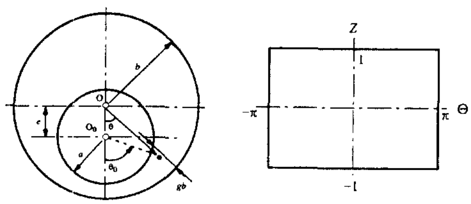


Fig. 1 Geometry of the annular space between two eccentric cylinders in the physical plane (r, θ) and in the computational domain (Z, Θ)

denoted by a, and b, respectively, as shown in Fig. 1. Annular gap between two eccentric cylinders is $G(=bg^{+}(\theta))$ and the eccentricity e. The annular gap is a function of eccentricity and azimuthal angle.

The axial pressure loss is constant in the fully developed laminar flow. In consideration of the characteristics of fully developed flow for convective heat transfer problem, the dimensionless temperature, $t^{+}=(t_w-t)/(t_w-t_m)$ based on mixed mean temperature defined later, are constant with axial direction. Variations of mixed mean temperature, t_m, and wall temperature, t_w, are linear with axial direction. Therefore, the governing equation including conduction heat transfer can be expressed in the form of a Poisson equation,

$$\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}+\frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}=\frac{1}{\mu}\frac{dp}{dx} \tag{5}$$

$$\frac{\partial^2 t}{\partial r^2}+\frac{1}{r}\frac{\partial t}{\partial r}+\frac{1}{r^2}\frac{\partial^2 t}{\partial \theta^2}=\frac{u}{\alpha}\frac{dt_m}{dx}-\frac{\dot{q}}{k} \tag{6}$$

where $\alpha(=k/\rho c)$ represents heat diffusion rate, $\dot{q}$ denotes heat source generated per unit volume. In the above equation, dt_m/dx is assumed to be constant and the heat source term is null for heat convection problem. The boundary conditions are

$$u\,|_{r=b-G}=0, \qquad u\,|_{r=b}=0 \tag{7}$$

$$t\,|_{r=b-G}=t_i, \qquad t\,|_{r=b}=t_o \tag{8}$$

where subscripts i and o stand for inner and outer cylinders, respectively.

The physical properties of steady fluid flow between two eccentric cylinders including concentric cylinder arrangements are a function of radial and circumferential coordinates. In order to generalize the problem by introducing a spectral collocation method based on the Chebyshev polynomials, it is convenient to transform the physical plane (r, θ) into the orthogonal plane of computational domain (Z, Θ) as shown in Fig. 1. For this purpose, the following coordinates with respect to origin of outer cylinder are defined as

$$Z=2\frac{r-(b-bg^{+})}{bg^{+}}-1, \quad \theta=\Theta \tag{9}$$

The coordinate transformation with respect to origin of inner cylinder instead of outer cylinder was introduced in the previous work (Mateescu, *et. al.,* 1994). For brevity, transformation of the governing equations to the origin of inner cylinder was not given here. In the present paper, an annular space G is defined as

$$G=bg^{+}(\theta)=b-e\cos\theta-aE^{1/2} \tag{10}$$

where E is a function of relative eccentricity, e/a, as

$$E=1-\left(\frac{e}{a}\right)^2\sin^2\theta \tag{11}$$

The application of chain rule of partial differentiation enables the transformation of partial differential equation from physical domain to computational domain. Considering the Eq. (9), the partial differentiations of variable, $f=f(Z, \Theta)$ in the radial direction becomes,

$$\frac{\partial f}{\partial r}=\frac{2}{bg^{+}}\frac{\partial f}{\partial Z}, \quad \frac{\partial^2 f}{\partial r^2}=\left(\frac{2}{bg^{+}}\right)^2\frac{\partial^2 f}{\partial Z^2} \tag{12}$$

The partial differentiations of nondimensional coordinate, $Z(r, \theta)$, to radial direction are written as

$$\frac{\partial Z}{\partial \Theta}=\frac{\partial Z}{\partial \theta}\frac{\partial \theta}{\partial \Theta}=-\frac{1-Z}{g^{+}}\frac{\partial g^{+}}{\partial \theta},$$

$$\frac{\partial^2 Z}{\partial \Theta^2}=\frac{\partial}{\partial \Theta}\left(\frac{\partial Z}{\partial \Theta}\right)$$

$$=-\frac{1-Z}{g^{+2}}\left(\frac{\partial g^{+}}{\partial \theta}\right)^2+\frac{1-Z}{g^{+}}\frac{\partial^2 g^{+}}{\partial \theta^2} \tag{13}$$

where

$$\frac{\partial g^{+}}{\partial \theta}=\frac{e}{b}\sin\theta+\frac{1}{2}\frac{a}{b}\left(\frac{e}{a}\right)^2\sin 2\theta E^{-0.5},$$

$$\frac{\partial^2 g^{+}}{\partial \theta^2}=\frac{e}{b}\cos\theta+\frac{a}{b}\left(\frac{e}{a}\right)^2\cos 2\theta E^{-0.5}$$

$$+\frac{1}{4}\frac{a}{b}\left(\frac{e}{a}\right)^4\sin^2 2\theta E^{-1.5} \tag{14}$$

Therefore, the partial differentiation of physical property with respect to circumferential direction in physical domain is now transformed into the partial differential equations in the computational domain, as

$$\frac{\partial f}{\partial \theta}=\frac{\partial f}{\partial \Theta}+\frac{1-Z}{g^+}\frac{\partial g^+}{\partial \theta}\frac{\partial f}{\partial Z},$$

$$\frac{\partial^2 f}{\partial \theta^2}=\frac{\partial^2 f}{\partial \Theta^2}+\left[\frac{1-Z}{g^+}\frac{\partial^2 g^+}{\partial \theta^2}-2\frac{1-Z}{g^{+2}}\left(\frac{\partial g^+}{\partial \theta}\right)^2\right]\frac{\partial f}{\partial Z}+2\frac{1-Z}{g^+}\frac{\partial g^+}{\partial \theta}\frac{\partial^2 f}{\partial Z \partial \Theta}+\left(\frac{1-Z}{g^+}\frac{\partial g^+}{\partial \theta}\right)^2\frac{\partial^2 f}{\partial Z^2} \quad (15)$$

With straightforward application of the conformal transformation, Eq. (9), and the partial differentiation, Eqs. (12) and (15), the governing equations become

$$A\frac{\partial^2 u}{\partial Z^2}+B\frac{\partial u}{\partial Z}+C\frac{\partial^2 u}{\partial Z \partial \Theta}+D\frac{\partial^2 u}{\partial \Theta^2}=\frac{G^2}{4\mu}\frac{dp}{dx} \quad (16)$$

$$A\frac{\partial^2 t^*}{\partial Z^2}+B\frac{\partial t^*}{\partial Z}+C\frac{\partial^2 t^*}{\partial Z \partial \Theta}+D\frac{\partial^2 t^*}{\partial \Theta^2}=-\frac{G^2}{4}\left[\frac{u}{a}\frac{dt_m}{dx}-\frac{\dot{q}}{k}\right] \quad (17)$$

where

$$A=1+D[(1-Z)G'/G]^2,$$
$$B=+\sqrt{D}+D(1-Z)[G''/G-2(G'/G)^2],$$
$$C=2D(1-Z)G'/G,$$
$$D=\{G/[2b-(1-Z)G]\}^2$$

In the above equations, prime and double prime denote $\partial/\partial\theta$ and $\partial^2/\partial\theta^2$, respectively. For the sake of simplification, the surface temperatures of two cylinders are assumed to be equal. Thus, the solution satisfies the following boundary conditions

$$u(1,\ \theta)=0 \qquad u(-1,\ \theta)=0$$
$$t^*(1,\ \theta)=0 \qquad t^*(-1,\ \theta)=0 \quad (18)$$

and $t^*=t_w-t,\ t_i=t_o=t_w$

Using spectral collocation method, the axial fluid velocity and temperature in annuli are expanded as

$$u(Z,\ \Theta)=\sum_{j=0}^{m}\sum_{k=0}^{n}U_{jk}C_j(Z)F_k(\Theta)$$

$$t^*(Z,\ \Theta)=\sum_{j=0}^{m}\sum_{k=0}^{n}\Gamma_{jk}C_j(Z)F_k(\Theta) \quad (19)$$

where $C_j(Z)$ and $F_k=\cos k\Theta$ stand for Chebyshev polynomials and Fourier function, respectively. The choice of Fourier series, $F_k=\cos k\theta$, for the interpolation functions in the circumferential direction stems from the obvious periodic character of the parameters with respect to and symmetric configurations.

Substituting Eq. (19) into Eq. (17), the energy equation is written as

$$\sum_{j=0}^{m}\sum_{k=0}^{n}\Gamma_{jk}[AC_j''(Z)F_k(\Theta)+BC_j'(Z)F_k(\Theta)+CC_j'(Z)F_k'(\Theta)+DC_j(Z)F_k''(\Theta)]=-\frac{G^2}{4}\left[\frac{1}{a}\frac{dt_m}{dx}\{\sum_{j=0}^{m}\sum_{k=0}^{n}U_{jk}C_j(Z)F_k(\Theta)\}-\frac{\dot{q}}{k}\right] \quad (20)$$

and the boundary conditions for the two cylinders at the identical temperature are

$$\sum_{j=0}^{m}\sum_{k=0}^{n}\Gamma_{jk}C_j(1)F_k(\Theta)=0$$
$$\sum_{j=0}^{m}\sum_{k=0}^{n}\Gamma_{jk}C_j(-1)F_k(\Theta)=0 \quad (21)$$

in terms of the unknown coefficients, Γ_{jk}.

Imposing the governing equation on $(m-1)$ collocation points equally distributed in the radial direction and $(n+1)$ points in the direction of circumference and considering the boundary condition, the algebraic equations of matrices are obtained, from which the *a priori* unknowns U_{jk} & Γ_{jk} can be determined. The number of collocation point in the radial and circumferential directions should be carefully selected to achieve the desired level of accuracy and, at the same time, good computing efficiency. The unknowns in the algebraic equation are solved by Gauss-Seidel iterating method based on pivot points, and then fluid velocity and temperatures are calculated. Now, it is not difficult to obtain the mean fluid velocity and the mixed mean temperature inside the annuli by integrating the equations as follows :

$$u_m=\frac{1}{\pi(b^2-a^2)}\int_o^{2\pi}\int_{r=b-G}^{r=b}urdrd\theta$$
$$t_m=\frac{1}{u_m\pi(b^2-a^2)}\int_o^{2\pi}\int_{r=b-G}^{r=b}utrdrd\theta \quad (22)$$

4. Friction Factor and Nusselt Number

Local wall shear stress acting on the wall is calculated by evaluating the velocity gradient at the wall, $\tau_{wall}=\pm\mu(\partial u/\partial n)_{wall}$ where n is the unit normal to the wall, and the average values of the wall shear stresses are given by

$$\tau_i)_{avg}=\overline{\tau_i}=\frac{1}{\pi}\int_o^{\pi}\tau_i d\theta_o,$$

$$\tau_i=\mu\frac{\partial u}{\partial n}\bigg|_i=\mu\frac{-2}{H}\frac{\partial u}{\partial Z}\bigg|_{Z=1} \tag{23}$$

$$\tau_o)_{avg}=\overline{\tau_o}=\frac{1}{\pi}\int_o^{\pi}\tau_o d\theta,$$

$$\tau_o=-\mu\frac{\partial u}{\partial n}\bigg|_o=\mu\frac{-2}{G}\frac{\partial u}{\partial Z}\bigg|_{Z=1} \tag{24}$$

where the integration is performed only over the range $0\le\theta\le\pi$ because of symmetry and H denotes annular gap at θ_o. In the above equations, the flow velocity gradient normal to the wall is calculated in the corresponding computational domain; *e. g.*, for the inner wall, the computational domain was defined using $Z=1-2(r-a)/H$, as shown in the previous work (Mateescu, *et. al.*, 1994). A friction factor, λ, is defined relative to both the inner wall and outer wall average shear stresses by the relations

$$\lambda_i=\frac{\overline{\tau_i}}{\rho u_m^2/2} \tag{25}$$

$$\lambda_o=\frac{\overline{\tau_o}}{\rho u_m^2/2} \tag{26}$$

where u_m is the mean velocity defined as in Eq. (22).

Generally, the average friction factor is defined as

$$\lambda=\frac{-(b-a)}{\rho u_m^2}\frac{dp}{dx} \tag{27}$$

where the pressure-loss-dependent mean flow velocity has been estimated by Eq. (22). From the overall force balance, the following relations are derived

$$\rho u_m^2(a\lambda_i+b\lambda_o)=-(b^2-a^2)\frac{dp}{dx} \tag{28}$$

Combining Eqs. (27) and (28) yields

$$\lambda Re=\frac{a\lambda_i Re+b\lambda_o Re}{a+b} \tag{29}$$

where Reynolds number is based on hydraulic diameter $D_h=2G$.

The transferred heat energy to the fluid per unit length through cylinder surface increases the mixed mean temperature and, in general, if no heat sources in the fluid and constant heat flux are assumed, the increasing rate of mixed mean temperature is constant. The energy conservation equation with respect to the unit cylinder length is written as

$$\dot{q}'=\dot{q}_i'+\dot{q}_o'=\pi(b^2-a^2)u_m\rho c\frac{dt_m}{dx}$$
$$=h2\pi(b+a)(t_w-t_m) \tag{30}$$

where $\dot{q}_i'$ and $\dot{q}_o'$ represent heat energy transferred through unit length of inner and outer cylinders, respectively, per unit time. By inspection of the above equation, it is true that the heat energy transferred per unit length through wall increases with mean flow velocity and the axial variation of mean temperature, which is constant in the present analysis.

The heat energy transferred through surface of cylinder can be determined by taking into account the normal temperature gradient on the cylinder surface. Thus, the heat energy transferred through unit length of outer cylinder, as an example, is evaluated by

$$\dot{q}_o'=-k\int_o^{2\pi}\frac{\partial t}{\partial n}\bigg|_{r=b}b d\theta=2\pi b h_o(t_w-t_m),$$

$$\dot{q}_o''(\theta)=-k\frac{\partial t}{\partial n}\bigg|_{r=b} \tag{31}$$

Finally, considering the convective heat transfer coefficient h obtained from the Eqs. (30) and (32), Nusselt number becomes

$$Nu=\frac{hD_h}{k}\ \text{where}\ h=\frac{bh_o+ah_i}{a+b} \tag{32}$$

5. Results and Discussion

It has been generally agreed that the approximate solutions obtained using spectral collocation method are accurate. In the study, however, in order to investigate the accuracy of the spectral collocation method in the heat transfer problem, the numerical results obtained by this approximate method are compared with the available exact solutions. In addition, the numerical results of Nusselt number are evaluated and are also compared with existing analytical results. The fluid considered in the present analysis is water. As generally understood, it is found that the numerical solution obtained by the spectral collocation method exponentially converges to exact solution with increasing the order of interpolation functions. The order of Chebyshev polynomials $C_j(Z)$ and Fourier function $F_k(\theta)$ applied in the present analysis are $m=7$ and $n=6$, respectively, and these numbers are found to be sufficient for the approximate solution with relatively good agreement between the numerical results and the analytical solutions.

The temperature gradient obtained by numerical method for the conductive heat transfer problem with uniform heat source is illustrated in Fig. 2 with a number of iso-temperature lines, where the surface temperatures of the hollow cylinders are $t_i=20$ ℃, $t_o=10$ ℃ and the heat source is $\dot{q}/k=4\times10^5$ ℃/m². The temperatures obtained numerically show a fairly good agreement with the analytical solution and the deviation is not visible in the Fig. 2 since its maximum deviation value is less than 10^{-4}. Therefore, the error through the whole calculation domain is extremely trivial. It is found that maximum values of temperature gradient and temperature exist at the widest region, $\theta=180°$, of the annular gap between two cylinders.

Numerical calculations of the present work, on the convective heat transfer problem generated by uniform heat flux through inner and outer cylinder walls, are achieved for $a/b=0.5$ (with $a=0.025$ m), $1/\mu\cdot dp/dx=-837.99$ (m·sec)$^{-1}$, $\alpha=1.342\times10^{-9}$ m²/sec and $dt_m/dx=0.5$ ℃/m with wall temperature $t_i=t_o$. As a pre-process result for convective heat transfer problem, iso-velocity lines are presented for $e/(b-a)=0.5$ in Fig. 3. With the calculated velocities, the nondimensional temperature, defined by $t^+=(t_w-t)/(t_w-t_m)$ based on the mixed mean temperature, is estimated by energy equation and nondimensional iso-temperature lines are illustrated in Fig. 4. Similarly as shown in the conductive heat transfer problem, it was found that the maximum temperature gradient and the maximum temperature appear at $\theta=180°$ where skin friction acting on the wall and the heat energy transferred through the wall are expected to have the maximum value on the inner cylinder. Fluid velocities and maximum temperatures obtained numerically are well consistent with the boundary conditions.

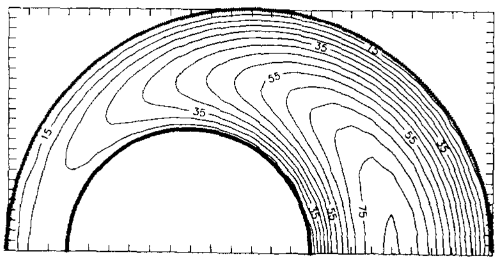


Fig. 2 Iso-temperature lines generated by uniform heat source for $a/b=0.5$, $e/(b-a)=0.5$, and $\dot{q}/k=4\times10^5$ ℃/m² with constant wall temperature $t_i=20$ ℃, $t_o=10$ ℃

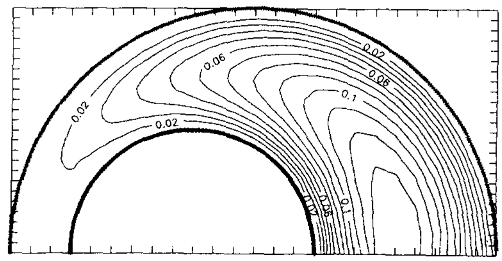


Fig. 3 Iso-velocity lines for $a/b=0.5$(with $a=0.025$ m), $e/(b-a)=0.5$ and $1/\mu\cdot dp/dx=-837.99$ (m·sec)$^{-1}$

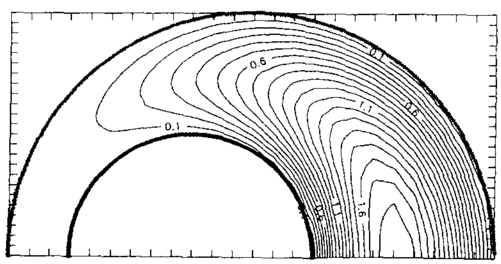
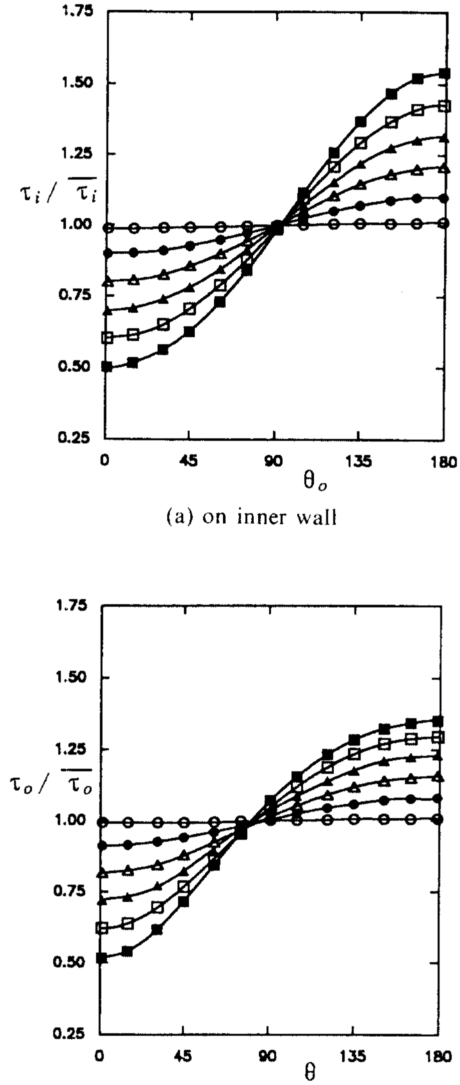


Fig. 4 Nondimensional iso-temperature lines generated by constant heat flux for $a/b=0.5$ (with $a=0.025$ m), $e/(b-a)=0.5$, $1/\mu\cdot dp/dx=-837.99$ (m·sec)$^{-1}$, $\alpha=1.342\times10^{-9}$ m²/sec and $dt_m/dx=0.5$ ℃/m with wall temperature $t_i=t_o$

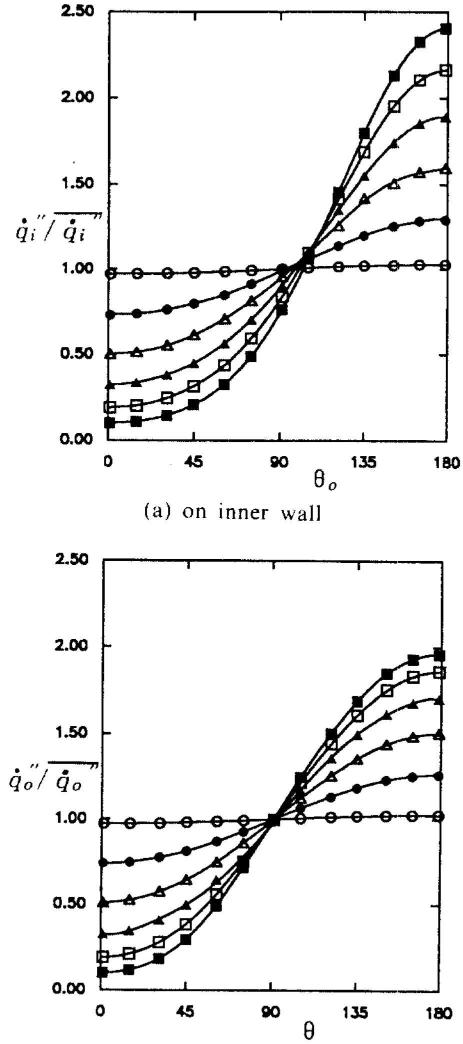


(a) on inner wall

(b) on outer wall

○, $e/(b-a)=0.01$; ●, $e/(b-a)=0.1$;
△, $e/(b-a)=0.2$; ▲, $e/(b-a)=0.3$;
□, $e/(b-a)=0.4$; ■, $e/(b-a)=0.5$

Fig. 5 Local shear stress rate, with respect to average values, at surface of cylinders along circumferential direction for $a/b=0.5$ (with $a=0.025$ m), $1/\mu \cdot dp/dx = -837.99$ $(\text{m} \cdot \text{sec})^{-1}$



(a) on inner wall

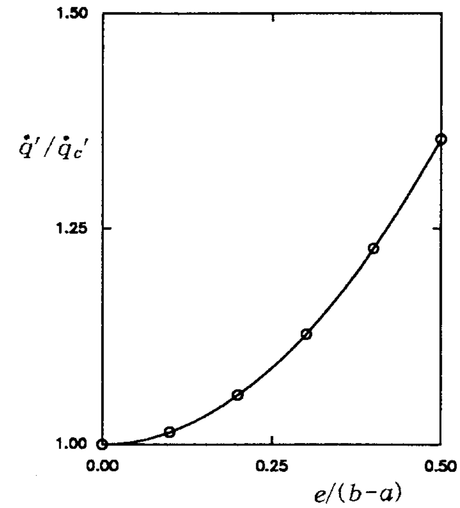
(b) on outer wall

○, $e/(b-a)=0.01$; ●, $e/(b-a)=0.1$;
△, $e/(b-a)=0.2$; ▲, $e/(b-a)=0.3$;
□, $e/(b-a)=0.4$; ■, $e/(b-a)=0.5$

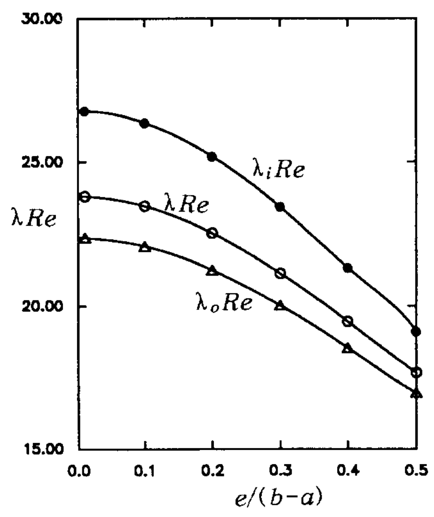
Fig. 6 Local heat transfer rate, with respect to average values, at surface of cylinders along circumferential direction for $a/b=0.5$ (with $a=0.025$ m), $1/\mu \cdot dp/dx = -837.99$ $(\text{m} \cdot \text{sec})^{-1}$, $\alpha = 1.342 \times 10^{-9}$ m^2/sec and $dt_m/dx = 0.5$ ℃/m with wall temperature $t_i = t_o$

Before presenting results of total friction factor and Nusselt numbers, local shear stress and heat transfer rate per unit area are evaluated with eccentricity and those rates with respect to the average values are plotted in Fig. 5 and in Fig. 6, respectively, with azimuthal angle. As predicted, the maximum values of the local friction factor and the heat transfer rate are shown at $\theta = 180°$ on inner cylinder. Variations of total friction factor and heat transfer rate with eccentricity are illustrated in Fig. 7. The subscript 'c' stands for concentric case in Fig. 7(a). With eccentricity, the friction factor decreases and the total heat transfer rate per unit length increases exponentially.

Total friction factors obtained by the present analysis are shown in Table 1 and compared with the analytical results. It is found that the maximum error with respect to analytical result is less than 10^{-3}. Nusselt numbers, calculated by the numerical method, are tabulated in Table 2. The calculated Nusselt numbers were compared with the available analytical results and it was found



(a)



(b)

Fig. 7 Variation of (a) total heat transfer rate with respect to average value of concentric case per unit length and, (b) friction factor on inner wall, outer wall and total friction factor, for $a/b=0.5$ with eccentricity

Table 1 Total friction factor, λRe, obtained by present numerical method with $m=7$ & $n=6$ and by existing analytical method(Snyder and Goldstein, 1965) in Bracket for eccentric annuli with uniform heat flux

a/b \ $e/(b-a)$	0	0.1	0.2	0.3	0.4
0.4	23.67 (23.68)	23.36 (23.36)	22.46 (22.47)	21.12 (21.13)	19.51 (19.52)
0.5	23.81 (23.81)	23.48 (23.48)	22.54 (22.54)	21.14 (21.14)	19.46 (19.46)
0.6	23.90 (23.90)	23.55 (23.56)	22.58 (22.59)	21.14 (21.14)	19.41 (19.42)
0.7	23.95 (23.95)	23.60 (23.60)	22.61 (22.62)	21.14 (21.15)	19.38 (19.39)
0.8	23.98 (23.98)	23.63 (23.63)	22.63 (22.63)	21.15 (21.15)	19.37 (19.37)

$\lambda Re=24$ (24) for $a/b=0.99$ and $e/(b-a)=0$

Table 2 Nusselt numbers obtained by present numerical method with $m=7$ & $n=6$ and by existing analytical method(Cheng and Hwang, 1968) in bracket for eccentric annuli with uniform heat flux

a/b \ $e/(b-a)$	0	0.1	0.2	0.3	0.4
0.4	8.034 (8.033)	7.561	6.496	5.394	4.511
0.5	8.117 (8.117)	7.619 (7.608)	6.473 (6.474)	5.316	4.393 (4.393)
0.6	8.170 (8.170)	7.635	6.453	5.254	4.309
0.7	8.203 (8.203)	7.664	6.437	5.212	4.249
0.8	8.223 (8.235)	7.665	6.426	5.184	4.211

$Nu=8.236(8.235)$ for $a/b=0.99$ and $e/(b-a)=0$

that the two results show a fairly good agreement. In particular, both numerical results, friction factor and Nusselt number, for very narrow annular gap were found to be very close to the solutions of the problems between two flat plates, which might be physically true.

6. Conclusions

A numerical study based on spectral collocation method has been performed on conduction and laminar forced heat convection in eccentric annuli. A simple but realistic model for hydrodynamically and thermally fully developed laminar flow with a uniform heat flux through each wall has been established. Total friction factor and Nusselt number have been obtained for various radius ratios of inner cylinder to outer ones with eccentricties. Using the same approach, the conductive heat transfer problems with uniform rate of internal heat generation in infinitely long hollow cylinder has been solved. This study undertakes to define the validity of the spectral collocation method for assessing heat transfer problems. To verify the present numerical method, the present results of Nusselt number and total friction factor are compared with the available analytical results. It is clear that there is a very good agreement between the two results.

The most important results under considerations, but generally known, are that (a) the circumferential variations of heat transfer rate and skin friction per unit area can be expressed in Fourier function, which implies the variations grow with eccentricity, (b) the maximum values of the local friction factor and the heat transfer rate exist at $\theta = 180°$ on inner cylinder, and (b) with eccentricity, the total friction factor decreases and the heat transfer rate per unit length increases exponentially. As a result, the eccentric configuration can be utilized as a thermal switch to increase the heat transfer rate with constant heat flux; since, the mean axial flow velocity increases with eccentricity for a given pressure loss per unit length.

The fundamental set of boundary conditions, *e. g.*, constant wall temperature, was not considered in the present study. It remains as a subject of future work involving a more complicated geometry, steep configuration and diffuser in annular passage.

References

Chang, S. H., Baek, W. P. and Lee, J. Y., 1994, "Proposed Concept of a Tube-Type Passive Water-Cooled Reactor without Emergency Core Cooling System," *Journal of the Korean Nuclear Society*, Vol. 26, No. 2, pp. 161~167.

Cheng, K. C. and Hwang, G.-J., 1968, "Laminar Forced Convection in Eccentric Annuli," *A. I. Ch. E. Journal*, Vol. 14, No. 3, pp. 510~512.

El-Saden, M. R., 1961, "Heat Conduction in an Eccentrically Hollow, Infinitely Long Cylinder with Internal Heat Generation," *Journal of Heat Transfer*, Vol. 83, pp. 510~512.

Macdonald, H. M., 1893, "On the Torsional Strength of a Hollow Shaft," *Camb. Phil. Soc.*, Vol. 8, pp. 62~68.

Marcus, P. S., 1984a, "Simulation of Taylor-Couette Flow. Part 1. Numerical Methods and Comparison with Experiment," *Journal of Fluid Mechanics*, Vol. 146, pp. 45~64.

Marcus, P. S., 1984b, "Simulation of Taylor-Couette Flow. Part 2. Numerical Results for Wavy-vortex Flow with one Travelling Wave," *Journal of Fluid Mechanics*, Vol. 146, pp. 65~113.

Mateescu, D., Paidoussis, M. P. and Sim, W. G., 1994, "A Spectral Collocation Method for Confined Unsteady Flows with Oscillating Boundary," *Journal of Fluids and Structures*, Vol. 8, pp. 157~181.

Piercy, N. A. V., Hooper, M. S. and Winny, H. F., 1933, "Viscous Flow through Pipes with Cores," *Phil. Mag.*, Vol. 15, No. 2, pp. 647~676.

Reynolds, W. C., Lundberg, R. E. and Mccuen, P. A., 1963, "Heat Transfer in Annular Passages," *Int. Journal Heat Mass Transfer*, Vol. 6, pp. 483~529.

Sim, W. G. and Cho, Y. C., 1993, "Study of Unsteady Fluid-Dynamic Forces Acting on a Flexible Cylinder in a Concentric Annulus," *KSME Journal*, Vol. 7, No. 2, pp. 144~157.

Sim, W. G., Han, K. N. and Hwang, S. T., 1995, "Dynamic Stability of a Flexible Cylinder in a Confined Inviscid Flow Based on Spectral Method," *The Fourth Pan American Congress of Applied Mechanics*, Buenos Aires, Vol. 2, pp. 365~371.

Snyder, W. T. and Goldstein, G. A., 1965, "An Analysis of Fully Developed Laminar Flow in an Eccentric Annulus," *A. I. Ch. E. Journal*, Vol. 11, No. 3, pp. 462~467.

Trombetta, M. L., 1971, "Laminar Forced Convection in Eccentric Annuli," *Int. Journal Heat Mass Transfer*, Vol. 14, pp. 1161~1173.